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On stability of the three 3-brane model

Debajyoti Choudhury, Dileep P. Jatkar, Uma Mahanta and Subrata Sur

Mehta Research Institute of Mathematics and Mathematical Physics
Chhatnag Road, Jhusi, Allahabad 211 019, India
E-mail: debchou@mri.ernet.in, dileep@mri.ernet.in,
mahanta@mri.ernet.in, subrata@mri.ernet.in

Abstract: We show that the Goldberger-Wise mechanism for the three 3-brane scenario proposed by Kogan et al. stabilizes the radion. We find that the system of 3-branes stabilizes in such a way that the loss in the scale factor is insignificant. That is, the negative tension brane chooses to stay close to the visible brane.

Keywords: Extra Large Dimensions, Beyond Standard Model, AdS-CFT Correspondance.

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Contents

1. Introduction

In the quest to explore physics beyond the Standard Model (SM), certain new proposals [1]–[3] have held considerable interest of the community. Amongst these is the proposal by Randall and Sundrum [2] wherein the SM fields live on one of the two 3-branes which are end of the world branes of a five dimensional spacetime. This scenario can be embedded in higher dimensional spacetime as well. One of the attractive features of this proposal is the resolution of the hierarchy problem between the Planck scale and the electroweak scale. This is achieved by choosing the geometry of the embedding spacetime with an exponential warp factor for the four dimensional spacetime which is the worldvolume of the 3-brane. This exponential warp factor produces a difference in the mass scales between the two end of the world 3-branes.

There are several variants of the Randall-Sundrum (RS) model involving multiple branes [4, 5], intersecting brane configurations [6] and supersymmetry [7]–[9]. Many of these models including the RS model have atleast one negative tension brane. This is demanded by the requirement of charge neutrality in the compact space. For example, in the original RS model, the visible world lives on the negative tension brane (i.e., the one with the induced cosmological constant $\Lambda_{\rm vis} < 0$).

Any proposal for physics beyond the SM has to pass very stringent tests laid down by the SM itself as well as by the cosmological observations that have been made till date. Brane world cosmology has been studied in refs. [10]–[18]. These papers conclude that the rate of expansion of the universe in the RS model is different from that of the familiar Friedman-Robertson-Walker (FRW) cosmology in four dimensions. In particular, in RS models, the Hubble parameter is proportional to the energy density ρ, i.e., $H \sim \rho$. In contrast, in our universe, the Hubble parameter seems to behave as $H \sim \sqrt{\rho}$. It is, however, possible to circumvent this problem if one realizes that the total energy density ρ is the sum of the vacuum energy density, i.e., the brane tension and the matter energy density ρ_m which lives on the brane. In this case, dependence of the Hubble parameter on the energy density is $H \sim \sqrt{\Lambda_{\rm vis}\rho_m}$.

This resolution brings up a new problem though. If the RS model is to solve the hierarchy problem, the visible brane needs to have a negative tension. It turns out that the Hubble parameter on such a brane could be real only if the matter energy density is negative. However, this would lead to antigravity in the brane world, a consequence clearly at variance with experimental observations.

Another issue which was not dealt with in the original RS proposal [2] was that of the stability of end of the world brane model. This was resolved by Goldberger and Wise (GW) [19].[1] They introduced a bulk scalar field into the model and showed that its (minimal) coupling to the bulk gravity stabilizes the two end of the world 3-branes at a critical distance r_c. For all parameters of $o(1)$ this can generate a warp factor of the order 10^{15} GeV thereby relating the Planck Scale to the SM scale.

Stabilization of the two branes has an additional and welcome consequence. In a series of recent papers [24], it has been shown that this procedure automatically cures the cosmological problem as well, albeit with minor modifications from the standard FRW scenario. However, it is still interesting to consider the possibility of our living on a positive tension brane.

Recently, Kogan et al. [25] proposed a modified version of the RS model which contains three 3-branes with both the end of the world branes (one at $y = 0$ and the other at $y = L_2$) being of positive tension and the third (moving) brane with negative tension (at $y = L_1$). We will call this the three 3-brane model. The visible world lives on one of the end of the world branes (at $y = L_2$). This construction automatically ensures that the cosmological constant on our brane is positive and thereby avoids the problem of the Hubble parameter. This model has an exponential warp factor between either of the positive tension branes and the negative tension brane. The phenomenological consequences are rather striking and have been studied in refs. [5, 26].

Stability of the Kogan et al. [25] proposal is quite crucial. This is because the moving brane has negative tension and we have exponential warp factors growing from the negative tension brane to end of the world positive tension branes. Gain in the scale due to the warp factor between the positive tension brane at $y = 0$ and the negative tension brane at $y = L_1$ is lost partially by the time it reaches our universe at $y = L_2$. It is easy to see that if this construction of three 3-brane system stabilizes for $L_1 \sim L_2/2$ then the loss in the scale hierarchy is near total. It is, therefore, crucial that $L_1/L_2 \lesssim 1$.

In the light of this, we take up the issue of stability of the Kogan et al. model [25] in this paper. We use the method of Goldberger and Wise [19] of coupling a bulk scalar field to the brane system. We find that the system does stabilize, and remarkably enough, for $L_1/L_2 \approx 1$. We also find that this result is not very sensitive to small changes in the mass m of the bulk scalar field as well as small fluctuations of the parameter k appearing in the warp factor as long as $m \ll k$.

[1] For other as well as related proposals of stabilization of the radion modulus see [9][20]–[23].

2. Stabilization of the three 3-brane model

In this section, we will first briefly review the Kogan et al. model [25] and will then proceed with the question of modulus stabilization. As mentioned in the introduction, the model of Kogan et al. contains three parallel 3-branes located at $L_0 = 0$, L_1 and L_2. The fifth dimension y has orbifold geometry $S^1/\mathbb{Z}_2$ and L_0 and L_2 are orbifold fixed points. The action for this configuration is

$$S = \int d^4x \int_{-L_2}^{L_2} dy \sqrt{G}\{2M^3 R - \Lambda_{\rm bulk}\} - \sum_i \int_{y=L_i} d^4x V_i \sqrt{-\hat{G}^{(i)}}\,, \tag{2.1}$$

where $\hat{G}^{(i)}_{\mu\nu}$ is the induced metric on the branes and V_i are their tensions. The five dimensional metric ansatz is given by

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2\,. \tag{2.2}$$

This metric preserves four dimensional Poincare invariance. The warp factor $\sigma(y)$ is just a constant conformal scale factor for the induced four dimensional metric. Although it is constant on a given 3-brane, its numerical value is different on different 3-branes.

For the three 3-brane model, the warp factor grows on either side of the negative tension brane up to the positive tension branes. The warp factor $\sigma(y)$ which solves the equations of motion obtained from eq. (2.1) is

$$\sigma(y) = kr_c(L_1 - ||y| - L_1|)\,, \tag{2.3}$$

when the brane tensions are $V_0 = -\Lambda/k$, $V_1 = \Lambda/k$ and $V_2 = -\Lambda/k$. Due to the warp factor, the physical mass M on the visible brane situated at $y = L_2$ is related to the naive mass parameter M_0 of the four-dimensional Minkowski theory by $M = WM_0$, where,

$$W = \exp\left(-kr_c(2L_1 - L_2)\right) \tag{2.4}$$

is the warp factor between the visible brane and the other end of the world brane. As mentioned earlier, to solve the hierarchy problem between the Planck scale and the SM scale, W should be of $o(10^{-15})$. Notice that in the three 3-brane model, if the moving brane located at $y = L_1$ stabilizes close to $L_1 = L_2/2$ then $W \sim 1$. In such a situation, we will not generate exponential scale factor in this model despite having the visible brane stabilized reasonably far from the other end of the world brane. Thus the ratio L_1/L_2 is as important, if not more, as the absolute stability of the three 3-brane system.

Before embarking on the task of stabilization of the three 3-brane model, let us note that the negative tension brane is classically unstable unless it is at an orbifold fixed point. We propose, hence, a slight modification of the Kogan et al. model, i.e., we consider the negative tension brane to be located at an orbifold fixed point at $y = L_1$. Ends of the world are still defined by the positive tension branes located at $y = 0$ and $y = L_2$. This just amounts to a simple translation on the orbifolded circle. Let us now consider coupling a bulk scalar field to this system of three 3-branes. Our technique is a straightforward generalization of the GW mechanism [19]. Consider the action for a bulk scalar field of mass m.

$$S_{\rm Bulk} = \frac{1}{2}\int d^4x \int_{-L_2}^{L_2} dy\sqrt{G}(G^{AB}\partial_A\chi\partial_B\chi - m^2\chi^2)\,, \tag{2.5}$$

where G_{AB} is the five dimensional metric given in eq. (2.2) with $\sigma(y)$ given in eq. (2.3) .

Following ref. [19], we impose the condition that the bulk scalar field satisfies certain boundary conditions at the location of each of the branes. The boundary potentials are

$$\begin{aligned} S_0 &= \int d^4x\lambda_0(\chi^2 - v_0^2)^2 \qquad \text{at } y = 0 \\ S_{L_1} &= \int d^4x\lambda_1(\chi^2 - v_1^2)^2 \qquad \text{at } y = L_1 \\ S_{L_2} &= \int d^4x\lambda_2(\chi^2 - v_2^2)^2 \qquad \text{at } y = L_2\,. \end{aligned} \tag{2.6}$$

We consider only those configuration of the bulk scalar which solve the equations of motion subject to the condition that the boundary potentials (at $y = 0$, L_1 and L_2) are minimised. This essentially amounts to neglecting dynamics of χ along the directions tangential to any of the 3-branes. This assumption is reasonable because we are looking only at the stability of the three 3-brane system at the moment and are not studying the phenomenologcal consequence of possible coupling of the bulk scalar field χ to matter fields living on the branes. It, therefore, suffices to concentrate on the equation of motion of χ only in y direction. Equation of motion for χ along y is

$$\partial_y^2\chi - 4\sigma'(y)\partial_y\chi - m^2r_c^2\chi = 0\,, \tag{2.7}$$

where $\sigma'(y) = d\sigma(y)/dy$. It is straightforward to find the solution to this equation and it is given by

$$\chi(y) = \exp\left(2\sigma'(y)y\right)\left[\tilde{A}\exp\left(\sigma'(y)\nu y\right) + \tilde{B}\exp\left(-\sigma'(y)\nu y\right)\right], \tag{2.8}$$

where, $\nu = \sqrt{4 + (mr_c)^2/\sigma'^2(y)}$, $(\tilde{A}, \tilde{B}) = (A, B)$ for $0 \le y \le L_1$ and $(\tilde{A}, \tilde{B}) = (C, D)$ for $L_1 \le y \le L_2$. It is worth mentioning here that the formal solution (2.8) is a function of $\sigma'(y)$ but within any given range of values of y, e.g. $0 \le y \le L_1$, $\sigma'(y)$ is independent of y. The coefficients A, B, C and D are determined by demanding that χ minimizes the boundary potential. This gives,

$$
\begin{aligned}
A &= \frac{v_0 - v_1 X^{2-\nu}}{1 - X^{-2\nu}}\,, & B &= \frac{v_1 X^{2-\nu} - v_0 X^{-2\nu}}{1 - X^{-2\nu}} \\
C &= \frac{v_1 X^{\nu-2} - v_2 Y^{\nu-2}}{X^{2\nu} - Y^{2\nu}}\,, & D &= \frac{-v_1 X^{\nu-2} Y^{2\nu} + v_2 X^{2\nu} Y^{\nu-2}}{X^{2\nu} - Y^{2\nu}}\,,
\end{aligned}
\tag{2.9}
$$

where, for notational convenience, we have made a change of variables from L_1 and L_2 to $X = \exp(-kr_cL_1)$ and $Y = \exp(-kr_cL_2)$. Substituting this solution into the action and integrating out y gives a four dimensional effective potential for X and Y. Writing this effective potential in terms of X and a new variable $R \equiv Y/X$, we have,

$$
\begin{aligned}
k^{-1}V(X,R) = {} & \frac{1}{1 - X^{2\nu}}\left[(\nu+2)(X^\nu v_0 - X^2 v_1)^2 + (\nu-2)(v_0 - X^{\nu+2} v_1)^2\right] + \\
& + \frac{X^4}{R^4(1 - R^{2\nu})}\left[(\nu+2)(R^2 v_1 - R^\nu v_2)^2 + (\nu-2)(v_2 - R^{\nu+2} v_1)^2\right].
\end{aligned}
\tag{2.10}
$$

An important thing to notice at this point is that, for arbitrary positive values of ν, the potential (2.10) grows as $X \to 1$ or as $R \to 1$ as long as $v_1 \neq v_0$ and $v_1 \neq v_2$. These two limits correspond to the negative tension brane approaching the positive tension brane at $y = 0$ and at $y = L_2$ respectively. In fact, the potential has the following singularity in these limits: $V(X,R) \sim (1 - X^{2\nu})^{-1} > 0$ as $X \to 1$ and $V(X,R) \sim (1 - R^{2\nu})^{-1} > 0$ as $R \to 1$. This implies that the negative tension brane experiences repulsive forces exerted on it by both the positive tension end of the world branes and thus the three 3-brane model cannot reduce to the two 3-brane model. In other words, for a fixed value of L_2, L_1 lies strictly inside the interval $(0, L_2)$. However, there is an interesting caveat in this and that has to do with the choice of vev's of the bulk scalar field on the 3-branes. If $v_1 = v_0$ and/or $v_1 = v_2$, then the leading singularity in the potential is removed and the subleading terms in the potential are attractive. In other words, for $v_1 = v_2$, the potential (2.10) is such that for R close to 1, V is attractive and assumes a finite value for $R = 1$. In this case, therefore, the two 3-brane limit is a stable one. The situation is similar, though not identical, for $v_1 = v_0$ and $X \to 1$. From here onwards we will work with the case when v_0, v_1 and v_2 take different numerical values. We still need to find out whether it is possible to stabilize L_2. An equally important question relates to the magnitude of $L_2 - L_1$. This number is important to get the correct metric scale factor on the visible brane.

We will first show how the three 3-brane system is stabilized by the bulk scalar field. In the process we will also be able to determine $L_2 - L_1$. Change of variables from X, Y to X and R has simplified the form of the potential (2.10) to a considerable degree, yet, minimization remains a complicated task. The first point to notice here is that the potential is bilinear in v_i and hence the results depend only on the ratios

$$
r_0 \equiv \frac{v_0}{v_1}\,, \qquad r_2 \equiv \frac{v_2}{v_1}\,. \tag{2.11}
$$

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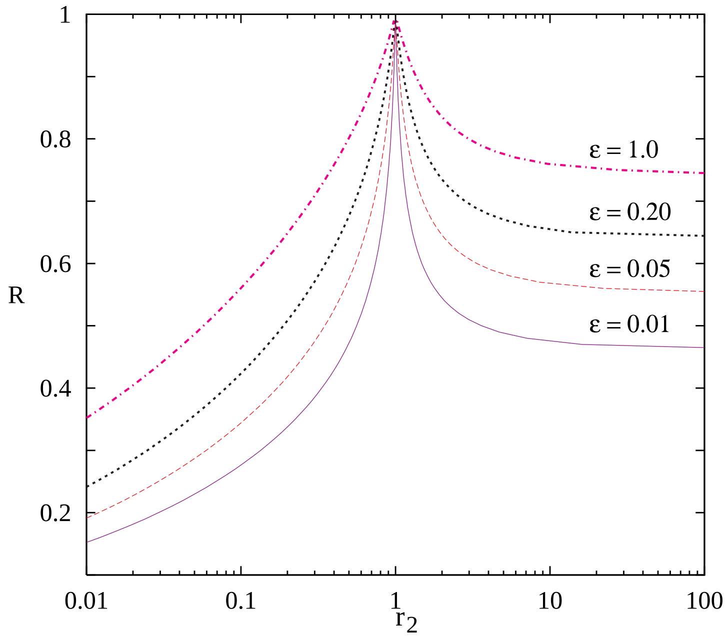

Figure 1: As the ratio r_2 of the vev of χ on the visible brane to that on the negative tension brane approaches 1, location of these two branes coincides. However, for $R < 1$ we have two possible values for r_2 as can be seen in this graph.

Furthermore, this choice of variables is such that the Hessian matrix is automatically diagonal on the extremal locus. Extremizing V with respect to R, we get

$$r_2^{\mp}(R) = \frac{\nu\, R^{2+\nu}\left[\left(2 \mp \sqrt{\nu^2 - 4}\right)\left(R^{2\nu} - 1\right) + \nu\,\left(1 + R^{2\nu}\right)\right]}{2\,\left(\nu^2\, R^{2\nu} + 2\left(R^{2\nu} - 1\right)^2 + \nu\,\left(R^{4\nu} - 1\right)\right)}\,. \tag{2.12}$$

An analytic inversion of this equation is not possible on account of its transcedental nature. Numerical solutions are straightforward though and we present the results in figure 1. For this purpose, instead of working with ν, it is instructive to consider

$$\epsilon \equiv \nu - 2 \approx \frac{m^2}{4k^2}\,, \tag{2.13}$$

with the approximate equality holding for a light bulk scalar field.

Although the solutions of eq. (2.12) exist for a wide range of ϵ, we are interested in small ϵ's and shall limit ourselves to this regime. In figure 1, the left and right branches correspond to r_2^- and r_2^+ respectively. At the cusp ($r_2 = 1$), the solutions move into unphysical domains. Looking at the figure, we surmise easily that $R \approx 1$ demands $r_2 \approx 1$. This is particularly pronounced for small ϵ.

While the solutions eq. (2.12) represent *minima* in the R-direction, it remains to be seen whether simultaneous minima in the X-direction exist. We answer this question next. Extremizing with respect to X, we obtain

$$r_0^\pm(X; R, r_2) = \frac{X^{2-\nu}}{2\,\nu} \left[2\left(1 - X^{2\,\nu}\right) + \nu \left(1 + X^{2\,\nu}\right) \pm \frac{X^{2+\nu}(1 - X^{2\nu})\sqrt{\mathcal{B}}}{\sqrt{1 - R^{2\,\nu}}} \right], \quad (2.14)$$

where

$$\mathcal{B} = \left(1 - R^{2\nu}\right)\left(\nu^2 + 4\nu - 12\right) - 8\nu + 16\nu R^{\nu-2} r_2 + 16 r_2^2 - 4 r_2^2 (\nu + 2)\left(R^{2\nu-4} + 1\right). \quad (2.15)$$

In each of eq. (2.14), r_2 assumes both of the values $r_2^\pm$. We thus have four possible branches in the solution space. Numerically though, for given R and X, the four solutions for r_0 roughly split into two pairs (the two r_0^-s on the one hand and the two r_0^+s on the other), with the intra-pair splitting considerably smaller than the inter-pair one. More interestingly, for a given ϵ,

- not the entire curve of figure 1 is admissible when confronted with eq. (2.14). Rather, only $r_2 \approx 1$ (and hence $R \approx 1$) can lead to a stable solution. For r_2 significantly different from unity, the quantity $\mathcal{B}$ in eq. (2.15) becomes negative resulting in unphysical values for X;
- for $r_0 = r_0^+$, $\partial^2 V/\partial X^2 < 0$ irrespective of the choice for r_2. Thus, each of these branches correspond to a sequence of saddle points.

In figure $2a$, we exhibit the relation between r_0 and r_2 that must be satisfied to obtain a particular warp-factor for a given ϵ. As is evident, W is a very sensitive function of r_0. This is not very unexpected as, for small ϵ, eq. (2.14) essentially gives r_0 to be polynomial function of $\ln X$ (and hence $\ln W$). This exponential dependence, a feature we share with the GW solution to the original RS model, could of course be termed a weakness of such stabilization schemes.

The dependence on ϵ — see figure $2b$ — is even more pronounced. This, again, is not unexpected as $\epsilon \sim\ o(1)$ implies $m \sim k$. For such large masses of a field propagating in the anti-de Sitter bulk, its wavefunction decays very fast. Consequently, its classical values at the two end of the world branes would be widely different.

3. Discussion

Brane world universe is one of the promising proposals for exploring the physics beyond the Standard model. There are several variants of the original proposal of Randall and Sundrum [2]. Many of them are invoked to aviod possible contradications with our existing knowledge of the Standard model physics and the standard big bang cosmology. The proposal of Kogan et al. [25] is along the same lines. We have shown in this paper that the three 3-brane model can be stabilized by coupling the system to a bulk scalar field and using the Goldberger-Wise formalism [19].

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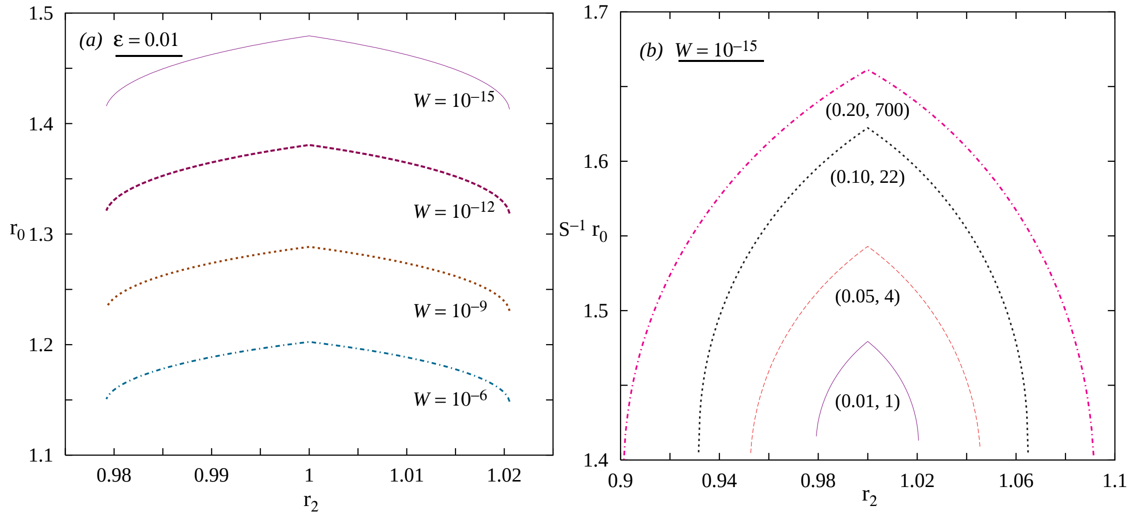

Figure 2: (a) For small values of ϵ, the warp factor is not very sensitive to the vev ratio r_2. The dependence on r_0 is rather pronounced though. Warp factor of 10^{-15} can be obtained with both the ratios r_0 and r_2 of o(1). (b) For small ϵ, a warp factor of 10^{-15} can be obtained with both the ratios r_0 and r_2 of o(1). However, for larger values of ϵ, the ratio r_0 is required to be quite large. The numbers in the parentheses refer to (ϵ, N) where N is the factor by which the y-axis has been rescaled for each curve.

We find that stability of the three 3-brane model is a more delicate problem than that for the Randall-Sundrum model. This is because gain in the scale due to the warp factor in the Randall-Sundrum model is partly offset by the moving brane. It is therefore not only necessary to have overall stabilization of the three 3-brane system but it is also necessary to have the ratio L_1/L_2 close to 1. We show that the generalization of the Goldberger-Wise mechanism to this model stabilizes the radion modulus in such a way that, for relatively small mass of the bulk scalar field or, equivalently, small ϵ, $L_1/L_2 \sim 1$. Consequently, it is possible to generate a scale hierarchy between the Planck scale and the TeV scale even without finetuning the parameters.

As we have discussed earlier, the modulus ceases to be stabilized in the event that the vevs on two (adjacent) branes are exactly equal. Such a degeneracy might seem natural, at least in the classical limit. However, it should be realized that quantum corrections are likely to disturb such an equality. This is particularly so since the vev on a brane is affected, at one loop level and higher, by the particle spectrum on the brane and their coupling to the bulk scalar field. Unless the spectrum is exactly alike on two adjacent branes, and unless their coupling to χ are exactly the same, it seems unlikely that the two vevs could be equal. And since it is only in this exactly equal vevs limit, that the potential becomes attractive, the point is, perhaps, a moot one.

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